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# HOLE–ELECTRON PRODUCT OF *pn* JUNCTIONS

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**Abstract**—Assertions that the hole–electron product may be larger than $n_i^2 \exp(qV/kT)$ under high injection conditions are found in the old and recent literature. This paper shows that the *np* product must always be $\leqslant n_i^2 \exp(qV/kT)$. Plots of numerical results for electrostatic potential, quasi-fermi levels and carrier concentration for an asymmetrical *np* junction under low and high injection conditions are given.

**Résumé**—Des indications que le produit trou-électron peut être supérieur à $n_i^2 \exp(qV/kT)$ à régime de fortes injection sont trouvées dans de vieux et récents articles. Cet article démontre que le produit *pn* doit toujours être $\leqslant n_i^2 \exp(qV/kT)$. Des courbes de résultats numériques de potentiel électrostatique, de niveaux quasi-Fermi et de concentration de porteur d'une jonction *np* asymétrique à basse et forte injection sont données.

**Zusammenfassung**—In der älteren und neueren Literatur wird behauptet, das Produkt aus Elektronen- und Löcherkonzentration könne bei starker Injektion grösser als $n_i^2 \exp(qV/kT)$ sein. In der vorliegenden Arbeit wird gezeigt, dass das *np*-Produkt immer $\leqslant n_i^2 \exp(qV/kT)$ sein muss. Numerisch berechnete Kurven für das elektrostatische Potential, Quasiferminiveaus und Trägerkonzentrationen werden für einen unsymmetrischen *np*-Übergang bei schwacher und starker Injektion gegeben.

RELATIONS are given in the old[1] and recent[2] literature which indicate that the hole–electron product in a *pn* junction under forward bias can be larger than $n_i^2 \exp(qV/kT)$, where $V$ is the applied junction voltage, or equivalently, that the separation of hole and electron quasi-fermi levels can be larger than the applied junction voltage.[3] In this paper we wish to reaffirm the generally accepted result[4] that under no circumstances (barring illumination with light, X-rays, etc.) can the *np* product exceed $n_i^2 \exp(qV_a/kT)$, where $V_a$, the applied voltage, is understood to be the difference between the majority carrier quasi-fermi levels at the contacts on either side of the junction. It is shown that the difficulties in Refs. 1 and 2 arise from relating applied voltage to the potential step across the junction under conditions where the majority carrier concentration differs from the equilibrium concentration on one or both sides. Finally, as an example, plots are given of numerical results for electrostatic potential, quasi-fermi levels, and carrier concentrations for an asymmetrical *np* junction under low and high injection conditions.

In 1957 FLETCHER[1] presented relations for the carrier concentrations just outside of the depletion region, from which one obtains for the *np* product on either side

$$\frac{np}{n_i^2} = \exp(qV/kT)\frac{1+(N_A/N_D+N_D/N_A)x+x^2}{1-x^2} \tag{1}$$

where $V$ is the "applied junction voltage", $x = \exp[q(V-\Phi)/kT]$, $\Phi = (kT/q)\ln(N_A N_D/n_i^2)$, and $N_A$ and $N_D$ are the net acceptor and donor concentrations. According to equation (1) the *np* product exceeds $n_i^2 \exp(qV/kT)$ for large forward bias for any $N_A$, $N_D$ combination.

The difficulty with equation (1) is contained in the phrase "applied junction voltage". In the context of equation (1), i.e. for high-level injection and in the immediate neighborhood of the depletion region, junction voltage cannot be defined as total potential step minus built-in voltage because, by definition, the majority carrier concentration on at least one side differs from its equilibrium concentration. If the points between which junction voltage is to be specified are moved far enough away from the junction so that a voltage is well defined in terms of the electrostatic potential step, then equation (1) is in error because of neglect of the consequences of the current flow that accompanies large forward bias, i.e. bending of the quasi-fermi levels.

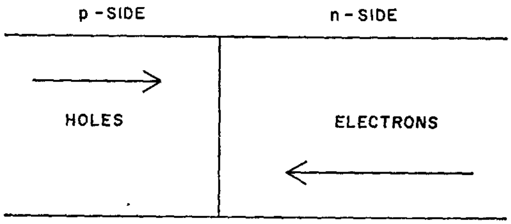


FIG. 1. Assumed directions of particle current flow.

The only consistent definition of applied voltage[5] is in terms of the difference of majority carrier quasi-fermi levels at the contacts. In a hypothetical situation of infinite lifetime and no recombination at the contacts, no currents would flow and the quasi-fermi levels would be constant, separated by the applied voltage. Then $np = n_i^2 \exp(qV/kT)$. If lifetime and recombination at the contacts are such that the majority carriers* have their equilibrium concentration at the contacts, as is usually the case, then the applied voltage is also equal to total potential difference minus zero-bias potential step, or built-in voltage. To explore the $np$ product for this case we consider a one-dimensional $pn$ diode, Fig. 1, and make the very plausible assumption that under forward bias the hole current is in the forward direction or, in the $n$-region, it may be zero; and similarly, that the particle current of electrons flows in the direction from the $n$-side to the $p$-side, or is zero. The contrary assumption that, say, at some point the hole current flows to the left would lead to severe inconsistencies.

* For present purposes the properties of the minority carriers at the contacts need not be considered.

As is well known, the hole and electron currents resulting from drift and diffusion can be expressed in terms of the gradient of the quasi-fermi levels $\varphi_p$ and $\varphi_n$:

$$j_p = -q\mu_p p \varphi_p' \quad (2)$$

$$j_n = -q\mu_n n \varphi_n' \quad (3)$$

where the as yet undefined symbols have their customary meaning. Since $j_p$, $q$, $\mu_p$ and $p$ are positive, the hole quasi-fermi level decreases monotonically to the right. Likewise the electron quasi-fermi level increases monotonically to the left (see Fig. 2). Thus, everywhere the separation of the quasi-fermi levels must be less than, or equal to, the applied voltage and therefore $np \leqslant n_i^2 \exp(qV/kT)$. Note that our argument does not depend on recombination in the depletion region. As long as recombination takes place *somewhere*, currents will flow.

In Ref. 2 quasi-fermi levels, or electrochemical potentials, are depicted which attain a separation larger than the applied voltage. These quasi-fermi levels give rise, however, to reverse majority currents at forward bias, a highly untenable situation.

As an illustration of a high-injection case we present plots of numerical results[6] for mid-band

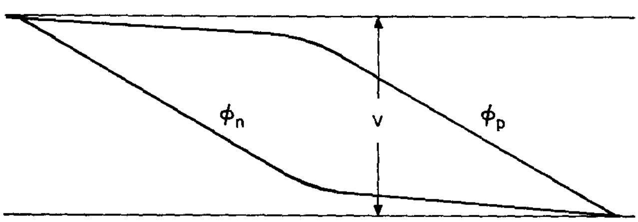


FIG. 2. Quasi-fermi levels for forward bias (schematic).

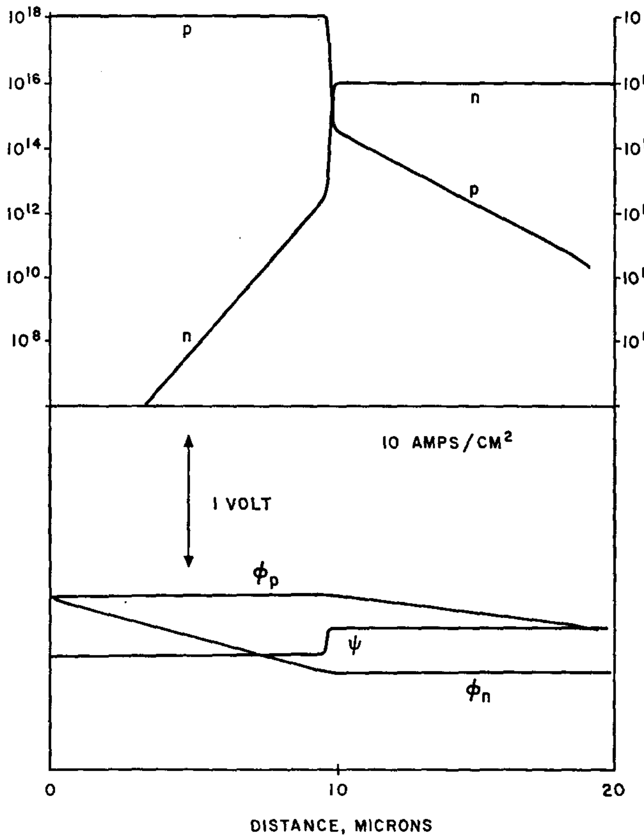


FIG. 3. Electrostatic potential, quasi-fermi levels and carrier concentrations at 10 A/cm$^2$.

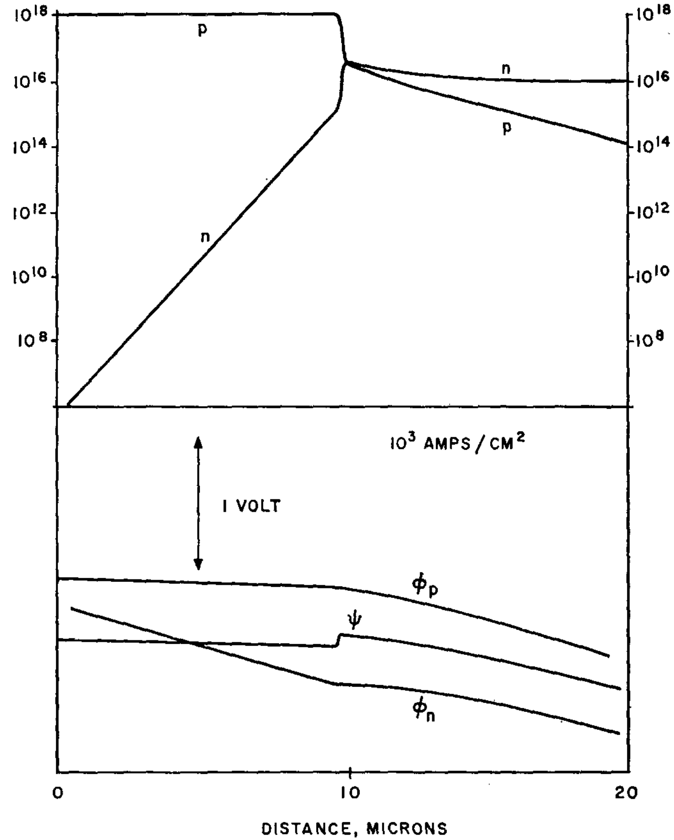


FIG. 4. Electrostatic potential, quasi-fermi levels and carrier concentrations at $10^3$ A/cm$^2$.

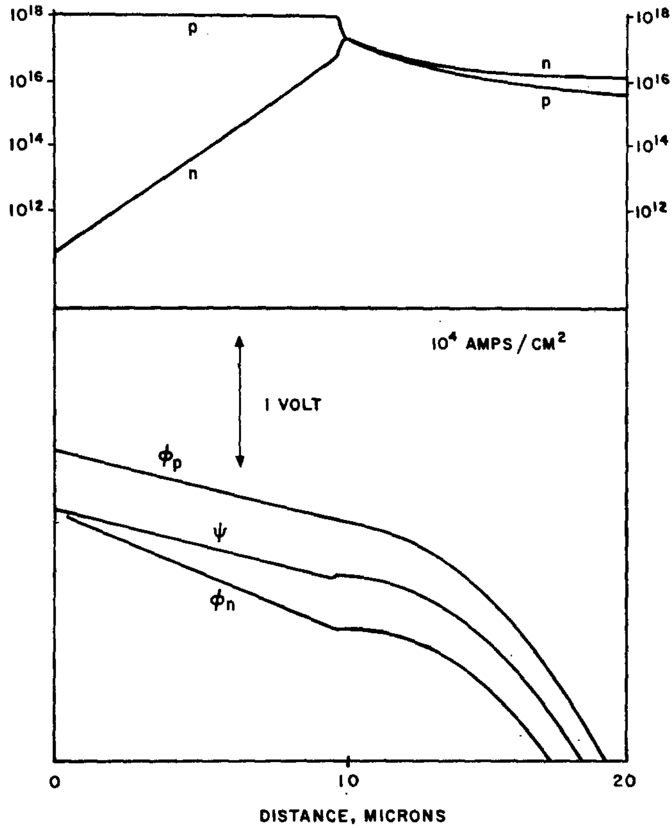


FIG. 5. Electrostatic potential, quasi-fermi levels and carrier concentrations at $10^4$ A/cm$^2$.

electrostatic potential, $\psi$, quasi-fermi levels, and carrier concentrations for a silicon *pn* step junction with the following parameters:

acceptor concentration $N_A = 10^{18}\ \text{cm}^{-3}$
donor concentration $N_d = 10^{16}\ \text{cm}^{-3}$
lifetime $\tau_n = 0.3 \times 10^{-9}$ sec
$\tau_p = 0.84 \times 10^{-9}$ sec.

The current densities in Figs. 3, 4 and 5 are $10^1$, $10^3$ and $10^4$ A/cm$^2$. At 10 A/cm$^2$ the diode is in the low-injection regime. Almost all of the potential drop occurs across the junction. The hole concentration on the *n*-side is small compared to the electron concentration. At $10^3$ A/cm$^2$ the electron concentration near the junction exceeds the donor concentration appreciably. An ohmic potential drop appears on the *n*-side. At $10^4$ A/cm$^2$ we have very heavy injection; the potential drop across the junction is insignificant in comparison to ohmic drops on both sides. Even though only the center region of the diode is shown in Figs. 3–5, it is apparent that the separation of the quasi-fermi levels at the junction is less than or equal to the difference of hole quasi-fermi level left of the junction and electron quasi-fermi level right of the junction for all forward bias levels.

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